DOCUMENT RESUME

ED 435 618		TM 030 215

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TITLE	Usefulness of the Logistic Positive Exponent Family of Models in Educational Measurement.
PUB DATE	1999-04-22
NOTE	29p.; Paper presented at the Annual Meeting of the American Educational Research Association (Montreal, Quebec, Canada, April 19-23, 1999).
PUB TYPE	Reports - Evaluative (142) -- Speeches/Meeting Papers (150)
EDRS PRICE	MF01/PC02 Plus Postage.
DESCRIPTORS	Cognitive Ability; *Educational Testing; Equations (Mathematics); *Measurement Techniques; Models; Personality Assessment; *Test Construction; Test Items
IDENTIFIERS	*Dichotomous Responses; Item Characteristic Function

ABSTRACT

The logistic positive exponent family (LPEF) of models has been proposed by F. Samejima (1998) for dichotomous responses. This family of models is characterized by point-asymmetric item characteristic curves (ICCs). This paper introduces the LPEF family, and discusses its usefulness in educational measurement and the implications of its use. Equations are given for the LPEF model. Two contrasting applications of the LPEF are discussed. One is an application in cognitive ability measurement for a situation in which the same task is assigned to two or more groups of individuals that differ in ability levels. With the LPEF, procedures of evaluation in each examination can be adjusted to suit each group. The other application is in personality or attitude measurement. Because models in the LPEF family are three-parameter models, it is advisable to use a nonparametric method to estimate the ICCs and then parameterize each of the resulting ICCs. (SLD)

USEFULNESS OF THE LOGISTIC POSITIVE EXPONENT FAMILY OF MODELS IN EDUCATIONAL MEASUREMENT[1]

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The 1999 Annual AERA Meeting

April 22, 1999

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I. Objective

The logistic positive exponent family (LPEF) of models has been proposed by Samejima (Psychometrika, 1998b) for dichotomous responses. While many mathematical models in the unidimensional item response theory are represented by point-symmetric item characteristic curve (ICC), or the conditional probability of the correct answer, given the latent trait θ, this family of models is characterized by point-asymmetric ICC's. It should be noted that the former group of models includes such popular models as the normal ogive model, the logistic model, Rasch model, the three-parameter logistic model, etc.

Although this family of models has been proposed, its implications and usefulness may not be very obvious to researchers in educational measurement. The objective of the present paper is to introduce the LPEF, and discuss its implications and usefulness in educational measurement.

II. Theoretical Framework: Logistic Positive Exponent Family of Models

Let θ be the latent trait, or *ability*, which assumes any real number, and g denote an item. The ICC of a model in the LPEF is given by

$$P_g(\theta) \equiv prob.[U_g = 1] = [\Psi_g(\theta)]^{\xi_g} \qquad \xi_g > 0 \ , \tag{1}$$

where U_g is a dichotomous item score which assumes either 0 (incorrect) or 1 (correct), and

$$\Psi_g(\theta) = \frac{1}{1 + \exp[-Da_g(\theta - b_g)]} \ , \tag{2}$$

where a_g is the discrimination parameter, b_g is the difficulty parameter, and $D(= 1.702)$ is the scaling factor. The third parameter, ξ_g, is called the acceleration parameter that characterizes this family of models.

seven examples whose ICC's were presented in Figure 1, and shown in Figure 2. Note that when $\xi_g = 1$, that is, in the logistic model, the IIF becomes a symmetric, unimodal curve, and, otherwise, those curves are unimodal but asymmetric, reflecting the fact that the ICC's are point-asymmetric when $\xi_g \neq 1$.

III. Implications of the LPEF Models

It is a common practice that researchers adopt a model that provides point-symmetric ICC's, which, for brevity, shall be called *symmetric* ICC's. One characteristic of a symmetric ICC is that it treats both correct and incorrect answers symmetrically. This leads to a logical contradiction in ordering examinees on the latent trait or ability scale.

Consider the maximum likelihood estimate (MLE) of the latent trait. For the purpose of illustration, following the normal ogive model, Table 1 presents the 32 possible response patterns of five dichotomous items that are arranged in the ascending order of the MLE's of the latent trait. These hypothetical items have a common discrimination parameter, $a_g = 1.0$, and separate, equally spaced difficulty parameters, $b_g = -3.0, -1.5, 0.0, 1.5, 3.0$, respectively. It can be seen by dividing the 32 response patterns into two subgroups, that is, the rows 1 through 16 and those 17 through 32, respectively, that the response patterns of the second group are compliments of those of the first group arranged in the reversed order.

Insert Table 1 About Here

It is logical to expect that the orders of MLE's are consistent for any pair of subsets of responses. Table 1 indicates, however, this consistency in rank order does not exist in the normal ogive model. If, for example, the response pattern with a subset 101 for items 2, 3 and 4 is ranked higher than the response pattern with another subset 110 for items 2, 3

It is obvious from Eqs. (1) and (2) that, when $\xi_g = 1$, the ICC in the LPEF becomes that of the logistic model. In this specific case, the ICC is represented by a *point-symmetric* curve, that is, the ICC has its point of symmetry at $(b_g, 0.5)$ and the relationship

$$P_g(\theta+) \;=\; 1 - P_g(\theta-) \quad , \tag{3}$$

holds with any real number d, where

$$\begin{cases} \theta+ \;=\; b_g \;+\; d \\ \theta- \;=\; b_g \;-\; d \;. \end{cases}$$

It should be noted that most mathematical models that have been widely used, such as the normal ogive model, the logistic model, Rasch model, 3-parameter logistic model, etc., provide *point-symmetric* ICC's. A strength of the LPEF is that the models provide point-asymmetric curves when $\xi_g \neq 1$ which do not satisfy Eq. (3), and enable them to order individuals on the latent trait dimension with a consistent philosophy.

Insert Figures 1 and 2 About Here

Figure 1 represents the ICC's of 7 examples in the LPEF given by Eqs. (1) and (2), with the common discrimination and difficulty parameters $a_g = 1$ and $b_g = 0$, and the separate acceleration parameters $\xi_g = 0.3, 0.5, 0.8, 1.0, 1.5, 2.0, 3.0$, respectively. The item information function (IIF) is given by

$$I_g(\theta) \;=\; \frac{[P_g'(\theta)]^2}{P_g(\theta)\;[1 - P_g(\theta)]} \tag{4}$$

for dichotomous response models in general, where $P'(\theta)$ indicates the first derivative of $P_g(\theta)$ with respect to θ. Substituting Eq. (1) and (2) into Eq. (4) the IIF's were obtained for the

and 4 in one environment, it is expected that the same rank order should exist in any other environments. Table 1 shows that, while the above rank order holds for the response patterns 01010 (#10) and 01100 (#6) and also for 01011 (#21) and 01101 (#19), the reversal of the rank order occurs for 11010 (#24) and 11100 (#25), and also for 11011 (#29) and 11101 (#30).

The same contradiction can be observed from another angle. It is noted in Table 1 that

1. The five response patterns, each of which contains only one correct response, are arranged in the order of difficulty of the item that is answered *correctly*, and
2. The five response patterns, each of which contains four correct responses are arranged in the order of difficulty of the item that is *not* answered correctly.

These two principles are contradictory to each other. If the first principle is accepted, then we should expect that, out of the five response patterns that have four correct answers each, the response pattern with the four most difficult items answered correctly to receive the highest ability estimate. However, if the second principle is true, then we should expect that, out of the five response patterns that have only one correct answer each, the response pattern with the easiest item answered correctly to receive the highest rank.

The above are just two examples, but the reversal of the two principles in assigning MLE's is seen in other response patterns also. These contradictions are intrinsic in all symmetric ICC's, with the exception of the logistic model, in which the MLE is not affected by the difficulty parameters, b_g 's for $g = 1, 2, ..., n$ (see Table 1). The contradiction in the rank order of response patterns does not exist in models of the LPEF, that provide asymmetric ICC's except for $\xi_g = 1$, however.

It is noted in Figure 1 that when $\xi_g < 1$ the ICC assumes higher values than the logistic ICC for the entire range of θ , and enhancement becomes larger as ξ_g gets less. Since Eq. (1)

can be written as

$$P_g(\theta) \ = \ [\Psi_g(\theta)]^{\xi_g} \ = \ \Psi_g(\theta) \ + \ [\{\Psi_g(\theta)\}^{\xi_g - 1} - 1] \ \Psi_g(\theta) \quad 0 \ < \ \xi_g \ < \ 1 \ , \tag{5}$$

$[\{\Psi_g(\theta)\}^{\xi_g-1} - 1]\ (> 0)$ can be considered as the conditional *elevation ratio*, given θ , which is strictly decreasing in θ and also strictly decreasing in ξ_g . In other words, if an item has an ICC given by Eq. (1) with very small positive ξ_g , then even individuals on very low ability levels have substantially high probabilities to *pass* the item. Thus it will be a natural expectation that, when a test consists of items with common a_g and $\xi_g\ (< 1)$ and different b_g 's , principle of *penalizing failure* in solving easier items should be consistently followed. This is confirmed by the examples illustrated in Table 2, in which $\xi_g = 0.3, 0.5, 0.8$.

Insert Table 2 About Here

It is a logical consequence that, for the same response pattern, the values of MLE are different, depending on the values of ξ_g 's ; for a smaller ξ_g the value of MLE is lower. This is well illustrated in Table 2. For example, for the response pattern 10111 the values of MLE are -0.81381 , 0.76848 and 1.89136 for $\xi_g = 0.3, 0.5, 0.8$, respectively. Note that all these values of MLE are less than 2.28753 , the value of MLE when $\xi_g = 1.0$, i.e., in the logistic model (see Table 1).

When $\xi_g > 1$, the ICC's assume lower values than the logistic ICC for all θ , as are illustrated in Figure 1 for $\xi_g = 1.5, 2.0, 3.0$. Since Eq. (1) can be rewritten in the form

$$P_g(\theta) \ = \ [\Psi_g(\theta)]^{\xi_g} \ = \ \Psi_g(\theta) \ - \ [1 - \{\Psi_g(\theta)\}^{\xi_g - 1}] \ \Psi_g(\theta) \quad \xi_g \ > \ 1 \ , \tag{6}$$

$[1 - \{\Psi_g(\theta)\}^{\xi_g-1}]\ (> 0)$ can be considered as the conditional *drop ratio*, which is strictly decreasing in θ and strictly increasing in ξ_g . In other words, if an item has an ICC given by Eq. (1) with large positive ξ_g , then even individuals with very high ability levels have a

substantially low probability to *pass* the item. Thus it will be reasonable to expect that when a test consists of items with common a_g and $\xi_g\ (> 1)$ and different b_g 's , the philosophy of giving credits to the success in solving more difficult items should consistently hold. This principle is confirmed by the examples in Table 3, in which $\xi_g = 1.5, 2.0, 3.0$.

Insert Table 3 About Here

As is the case with $\xi_g < 1$, it is a logical consequence that, for the same response pattern, the values of MLE are different depending on the values of ξ_g . Again for a smaller ξ_g the value of MLE is lower, as illustrated in Table 3. For example, for the same response pattern 10111 that was illustrated earlier, the values of MLE are 2.84408 , 3.14744 and 3.50199 for $\xi_g = 1.5, 2.0, 3.0$, respectively. Note that all these values of MLE are higher than those three counterparts for $\xi_g = 0.3, 0.5, 0.8$ and also the value of MLE in the logistic model.

The logistic model that is obtained by setting $\xi_g = 1$ in Eq. (1) can be interpreted, therefore, as a *transition* between the two opposing principles in the LPEF, and in this specific case both principles are degenerated. Thus item difficulties will not affect the order of MLE's.

IV. Usefulness of the LPEF Models

One concern may be in what cases the models in LPEF should appropriately be adopted. In this section, two contrasting applications of the LPEF will be given and discussed. It is hoped that readers will use them as hints, expand their imaginations, use analogies, etc., in order to find a use for LPEF models in their own research.

[IV.1] An Application in Cognitive Ability Measurement

Suppose there are two training programs for a certain computer language. In each program, the trainees' progresses are evaluated by having them write actual computer programs of the

same set of contents, using the language they have learned. In one training program, these exams are given with the instructor's simple and straight-forward explanations of the content of the target computer program, and the trainees are supposed to write a computer program on their own. When each trainee decides that his/her program should run correctly, it will be handed in. In the other training program, the trainees are allowed to use the programs they have written with data to find out if they actually run, and if the programs do not run well he/she can trouble-shoot and modify it up to, say, five times, and then the printout after the fifth revision should be handed in.

Because the content of each computer program has its own difficulty level, it will be represented by its difficulty parameter. Since the evaluation procedures in the two training programs are substantially different for the same contents of exams the values of the acceleration parameter should be expected to be different for the two different training programs.

It should be noted that, in the first training program, the trainees must take all factors into consideration and produce a correct computer program in the first trial without any feedback information. Thus only trainees who have very high programing ability have a high probability to pass the exam, and passing the exam deserves high credit. Thus an LPEF model with $\xi_g > 1$ will fit. In the second training program, since the trainees are allowed to make mistakes, trouble-shoot and make revisions up to five times, even those on relatively lower levels of ability will have a high probability to pass the exam. Thus penalization of the failure in writing a useable program should be emphasized. An LPEF model with $0 < \xi_g < 1$ will be suitable in such a case.

Usefulness of LPEF models is pronounced in this example in the sense that, when the same task is assigned to two or more groups of individuals that differ in ability levels, procedures of evaluation in each exam can be adjusted to suite each group. These different instructions will affect the parameter ξ_g for each group of individuals. Note that the same response pattern for the same set of items will not provide the same MLE for the two or more training programs

as was observed earlier, and yet these estimated ability levels of individuals in these separate programs can still be located on the same ability dimension.

For example, if there are five tests in the training programs and the acceleration parameter assumes 2.0 in the first program and 0.5 in the second, the MLE will be 0.77745 in the first program for the *pass-fail* pattern of 00110 , while it will be −2.59861 in the second program for the same pass-fail pattern (see Tables 2 and 3). For the seven different values of the acceleration parameter, $\xi_g = 0.3, 0.5, 0.8, 1.0, 1.5, 2.0, 3.0$, that were cited earlier, the values of MLE for this specific pass-fail pattern are −3.62818 , −2.59861 , −1.39938 , −0.75260 , 0.24694 , 0.77745 and 0.1.33889 , respectively. It is noted that the MLE increases with ξ_g , and this relationship holds for any pass-fail pattern.

There is a possibility that this relationship between the pass-fail pattern and the MLE gives an unqualified disadvantage to a bright individual. Suppose that a bright individual is misclassified into the second training program, and this person's pass-fail pattern turned out to be 11110 . If $\xi_g = 0.5$ in the second program as was exemplified earlier, then his MLE will be 1.76665 . Suppose, further, that for items 1 through 4 this subject actually completed the computer programs without even running data to confirm that the programs were right. In such a case this individual would have got the same pass-fail pattern, 11110 , had he/she been put into the first training program where $\xi_g = 2.0$; and yet he/she will get unfairly low value of 1.76665 as his/her estimated ability level, instead of 2.76207 .

A solution for this problem will be the use of graded scores. For example, scores can be given in such a way that those who wrote a useable computer program:

1. on their own get score 6,
2. after one set of running data and trouble-shooting get score 5,
3. after two sets of the above process get score 4,
4. after three sets of the above process get score 3,
5. after four sets of the above process get score 2, and

6. after five sets of the above process get score 1,
7. those who failed in writing a useable computer program even after five sets of the above process get score 0.

Thus a LPEF model on the graded response level (Samejima, 1997) will be applied. In this way the possibility of unqualified disadvantage for bright individuals will disappear, and there is no need to use two separate training program either. A trade-off is that the evaluation process will become more complicated, and a stricter supervision by the tester will be needed.

[IV.2] An Application in Personality or Attitude Measurement

It is desirable that in any personality or attitude measurement that our inventory or questionnaire should measure a wide range of the latent trait accurately, whether it is a specific personality scale or an attitude scale toward a specific topic. This accuracy of measurement can be evaluated *locally* for each scale, or as a function of the latent trait θ . This is done by the use of the inverse of the square root of the test information function, $I(\theta)$, which is given by

$$I(\theta) = \sum_{g=1}^{n} I_g(\theta) \quad , \tag{7}$$

where $I_g(\theta)$ is the item information function provided by Eq. (4), as the local standard error of estimation.

For the purpose of illustration, Figures 3 presents the test information function for each of the seven hypothetical tests (or inventories or questionaires). Each test consists of thirteen dichotomous items, with a common discrimination parameter $a_g = 1$, and the difficulty parameter b_g varies from -3 to +3 with the interval width of 0.5 . The acceleration parameter ξ_g varies for separate tests, and they are $0.3, 0.5, 0.8, 1.0, 1.5, 2.0$ or 3.0 , respectively.

Insert Figures 3 and 4 About Here

It is obvious from Figure 3 that, except for the lower range of θ, the amount of information becomes larger when the acceleration parameter ξ_g is higher. Actually these discrepancies are a little exaggerated, for it is not $I(\theta)$ but its square root that is counted. Figure 4 presents $\sqrt{I(\theta)}$ of the same seven hypothetical tests.

The local standard error of estimation, $[I(\theta)]^{-1/2}$, for each the same seven hypothetical tests is presented as Figure 5. This figure is informative. For example, approximating the conditional distribution of MLE, given θ, by the normal distribution with the mean θ and the standard deviation $[\sqrt{I(\theta)}]^{-1}$, the 68 percent confidence interval at $\theta = 2.0$ is $(1.53, 2.47)$, while it is $(1.11, 2.89)$ when $\xi_g = 0.3$, indicating that in the latter case estimation of θ is less accurate than in the former. The relative widths of the confidence intervals are reversed at, say, $\theta = -3.5$ where they are $(-5.02, -1.98)$ and $(-4.41, -2.59)$, respectively.

Insert Figure 5 About Here

Observations that were made above indicate that, in order to measure the latent trait reasonably accurately for a wide range of θ it will be desirable to mix items with varieties of different values of ξ_g. To realize this, we must look into the items to see if there is a possibility to adjust the value of ξ_g.

Take the Minnesota Multiphasic Personality Inventory (MMPI) as an example. MMPI basically consists of ten personality scales, such as depression, schizophrenia, social introversion, etc., and each scale has its own set of statements or items. Each statement is written as a first-person singular sentence, and the examinee is expected to answer these questions either "true" or "false" (with an additional category of "cannot say"). Consider the following four example statements (Rogers, T. B., 1995):

1. I am concerned about sex matters.

2. Some of my family have habits that bother me very much.

3. It takes a lot of argument to convince most people that they are wrong.

4. I wish I were not as shy as I am.

It will be reasoned that if we change item 3 to the sentence:

(3a) It takes some argument to convince most people that they are wrong,

the ICC will be changed also, and most likely the value of ξ_g becomes less, inviting more individuals on lower levels of θ to answer "true." This will also be the case with item 1, and if it is changed to:

(2a) Some of my family have habits that bother me,

the value of ξ_g will be shifted in the same direction. On the other hand, if item 1 is changed to:

(1a) I am concerned about sex matters very much,

then the value of ξ_g will become higher. These predictions will be confirmed or disconfirmed by estimating the ICC's of both the original and revised items in appropriate pilot studies, and comparing the two resultant estimates of ICC. It can be seen that such modifications are possible with many items in personality or attitute measurement. In contrast, item 4 may not have room for modification as the other three items do. It should be expected, therefore, that modifications are not possible for all items. If a large number of statements have room for modification, then it will be possible to modify or develop an inventory that has a sufficiently small and practically constant standard error of estimation over a wide range of θ .

V. Conclusions and Scientific Importance

Models in the LPEF are three-parameter models, so it is advisable to use a *nonparametric*

method (e.g., Samejima, 1998a) for estimating ICC's, and then parameterize each of the resulting ICC's. This procedure will ameliorate indeterminancy of the parameter estimates that is unavoidable when the model contains more than two parameters.

There is a gap between psychometricians who actively propose new mathematical models and researchers who apply mathematical models in educational measurement, and thus valid mathematical models are often overlooked by the latter group of researchers. Since mathematical models are useless unless they are validly used in empirical research, including educational measurement, explanations of the natures, implications and usefulness of a specific model will be important. The proposed paper is believed to have scientific importance in this regard.

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AERA992.TEX

March 24, 1999

TABLE 1

MLE's of θ Based on 32 Response Patterns of 5 Dichotomous Items Following the Normal Ogive Model and the Logistic Model with the Item Parameters $a_g = 1.0$ for All Items and $b_g = -3.0, -1.5, 0.0, 1.5, 3.0$, Respectively, Arranged in the Ascending Order of Those in the Normal Ogive Model.

	RESPONSE PATTERN	NORMAL OGV.	LOGISTIC
1	00000	NEG.INFINITY	NEG.INFINITY
2	10000	-2.28385	-2.28753
3	01000	-2.27016	-2.28753
4	00100	-1.84831	-2.28753
5	00010	-1.34811	-2.28753
6	01100	-1.15759	-0.75260
7	00001	-0.86577	-2.28753
8	11000	-0.75034	-0.75260
9	10100	-0.75021	-0.75260
10	01010	-0.75013	-0.75260
11	00110	-0.75011	-0.75260
12	00101	-0.36062	-0.75260
13	10010	-0.34310	-0.75260
14	01001	-0.27309	-0.75260
15	00011	-0.19116	-0.75260
16	01110	-0.15292	0.75260
17	10001	0.15292	-0.75260
18	00111	0.19116	0.75260
19	01101	0.27309	0.75260
20	10110	0.34310	0.75260
21	01011	0.36062	0.75260
22	10011	0.75011	0.75260
23	10101	0.75013	0.75260
24	11010	0.75021	0.75260
25	11100	0.75034	0.75260
26	01111	0.86577	2.28753
27	11001	1.15759	0.75260
28	10111	1.34811	2.28753
29	11011	1.84831	2.28753
30	11101	2.27016	2.28753
31	11110	2.28385	2.28753
32	11111	POS.INFINITY	POS.INFINITY

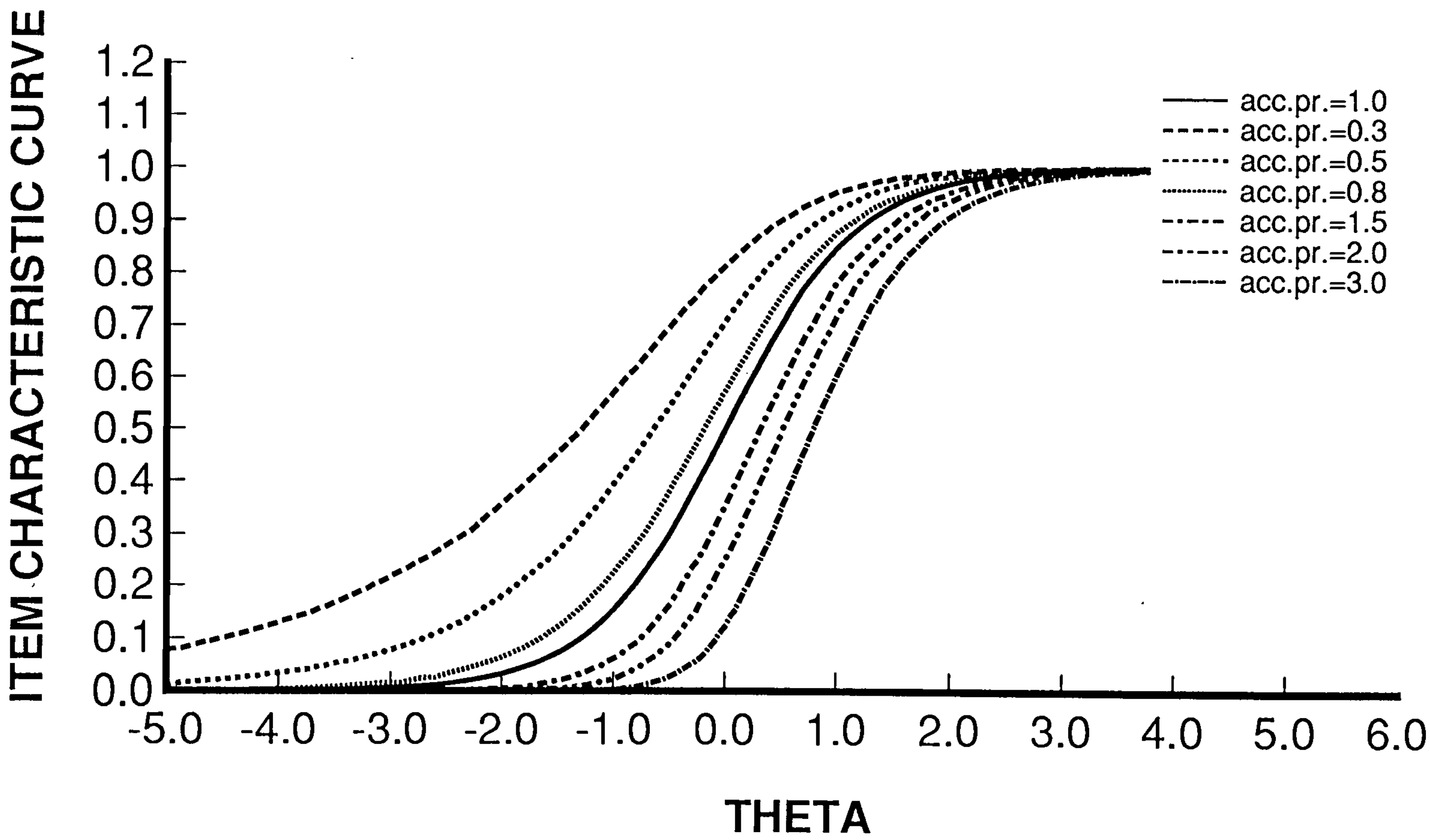
TABLE 2: MLE's of θ in 3 LPEF Models with $0 < \xi_g < 1$, Based on 32 Response Patterns of 5 Dichotomous Items with $a_g = 1.0$ and $b_g = -3.0, -1.5, 0.0, 1.5, 3.0$, Respectively, Arranged in the Ascending Order of the MLE's in Each Model.

		$\xi_g = 0.3$			$\xi_g = 0.5$			$\xi_g = 0.8$
1	00000	neg.infinity	1	00000	neg.infinity	1	00000	neg.infinity
2	00001	-4.77453	2	00001	-3.63928	2	00001	-2.75505
3	00010	-4.74061	3	00010	-3.62863	3	00010	-2.75233
4	00100	-4.66044	4	00100	-3.58957	4	00100	-2.73539
5	01000	-4.44706	5	01000	-3.43918	5	01000	-2.64931
6	00011	-3.73122	6	10000	-2.94596	6	10000	-2.47686
7	10000	-3.68305	7	00011	-2.68315	7	00011	-1.47400
8	00101	-3.65918	8	00101	-2.61820	8	00101	-1.41280
9	00110	-3.62818	9	00110	-2.59861	9	00110	-1.39938
10	01001	-3.44227	10	01001	-2.35019	10	01001	-1.25459
11	01010	-3.40499	11	01010	-2.32052	11	01010	-1.23813
12	01100	-3.30901	12	01100	-2.20138	12	10001	-1.19208
13	00111	-2.96638	13	10001	-1.96325	13	10010	-1.17564
14	10001	-2.74156	14	10010	-1.92990	14	01100	-1.15416
15	10010	-2.68833	15	10100	-1.79829	15	10100	-1.09344
16	01011	-2.64395	16	00111	-1.66989	16	11000	-0.93537
17	10100	-2.53970	17	11000	-1.37164	17	00111	0.01450
18	01101	-2.47870	18	01011	-1.22332	18	01011	0.10113
19	01110	-2.40218	19	10011	-1.05850	19	10011	0.11064
20	10011	-1.99519	20	01101	-0.92349	20	01101	0.26384
21	11000	-1.97075	21	01110	-0.79538	21	10101	0.27126
22	10101	-1.78917	22	10101	-0.78363	22	11001	0.33298
23	10110	-1.69818	23	10110	-0.66364	23	01110	0.34880
24	01111	-1.18201	24	11001	-0.40558	24	10110	0.35575
25	11001	-1.11762	25	11010	-0.27237	25	11010	0.41578
26	11010	-0.96788	26	11100	0.15865	26	11100	0.57391
27	10111	-0.81381	27	01111	0.75388	27	01111	1.89079
28	11100	-0.38312	28	10111	0.76848	28	10111	1.89136
29	11011	-0.11965	29	11011	0.89590	29	11011	1.89818
30	11101	0.65101	30	11101	1.29471	30	11101	1.95895
31	11110	1.32328	31	11110	1.76665	31	11110	2.12651
32	11111	pos.infinity	32	11111	pos.infinity	32	11111	pos.infinity

TABLE 3: MLE's of θ in 3 LPEF Models with $1 < \xi_g$, Based on 32 Response Patterns of 5 Dichotomous Items with $a_g = 1.0$ and $b_g = -3.0, -1.5, 0.0, 1.5, 3.0$, Respectively, Arranged in the Ascending Order of the MLE's in Each Model.

		$\xi_g = 1.5$			$\xi_g = 2.0$			$\xi_g = 3.0$
1	00000	neg.infinity	1	00000	neg.infinity	1	00000	neg.infinity
2	10000	-1.97543	2	10000	-1.77109	2	10000	-1.49830
3	01000	-1.72256	3	01000	-1.40646	3	01000	-1.03844
4	00100	-1.40982	4	00100	-0.83936	4	00100	-0.22596
5	00010	-1.29107	5	00010	-0.44235	5	11000	0.02377
6	00001	-1.27506	6	00001	-0.34778	6	10100	0.46812
7	11000	-0.44747	7	11000	-0.24612	7	01100	0.49242
8	10100	-0.20737	8	10100	0.10334	8	00010	0.60095
9	01100	-0.18094	9	01100	0.13035	9	00001	1.20399
10	10010	0.09996	10	10010	0.66449	10	10010	1.27538
11	01010	0.11753	11	01010	0.67548	11	01010	1.28060
12	10001	0.21808	12	00110	0.77745	12	00110	1.33889
13	01001	0.23485	13	10001	1.06032	13	11100	1.52543
14	00110	0.24694	14	01001	1.06796	14	11010	1.96863
15	00101	0.36499	15	00101	1.14590	15	10110	1.99289
16	00011	0.67782	16	11100	1.25580	16	01110	1.99471
17	11100	1.05493	17	00011	1.47795	17	10001	2.10133
18	11010	1.29427	18	11010	1.60421	18	01001	2.10278
19	10110	1.32062	19	10110	1.63116	19	00101	2.12062
20	01110	1.32270	20	01110	1.63323	20	00011	2.27531
21	11001	1.60183	21	11001	2.16546	21	11001	2.77572
22	10101	1.61938	22	10101	2.17644	22	10101	2.78094
23	01101	1.62074	23	01101	2.17729	23	01101	2.78134
24	10011	1.74879	24	10011	2.27846	24	10011	2.83927
25	01011	1.74974	25	01011	2.27907	25	01011	2.83961
26	00111	1.76139	26	00111	2.28672	26	00111	2.84388
27	11110	2.56892	27	11110	2.76207	27	11110	3.02705
28	11101	2.81505	28	11101	3.11779	28	11101	3.47544
29	11011	2.84197	29	11011	3.14533	29	11011	3.49999
30	10111	2.84408	30	10111	3.14744	30	10111	3.50199
31	01111	2.84425	31	01111	3.14760	31	01111	3.50214
32	11111	pos.infinity	32	11111	pos.infinity	32	11111	pos.infinity

SD~ASMICC95/P9601, 9601F, ICC OF 7 ITEMS, DC.RASCH.MDL, AG=1, BG=0.0, CG=0.3,0.5,0.8,1.0,1 .5,2.0,3.0; 9601RASCH.RST14; 3/06/96



0.500 1.00 1.50 6.00 8.00
9601RSC1.DAT, IN9601IC, plotted by M. Foster

FIGURE 1

Seven examples of the item characteristic curves of models in the logistic positive exponent family with the common discrimination parameter $a_g = 1$ and the common difficulty parameter $b_g = 0$, and the seven different acceleration parameters $\xi_g = 0.3, 0.5, 0.8, 1.0, 1.5, 2.0, 3.0$.

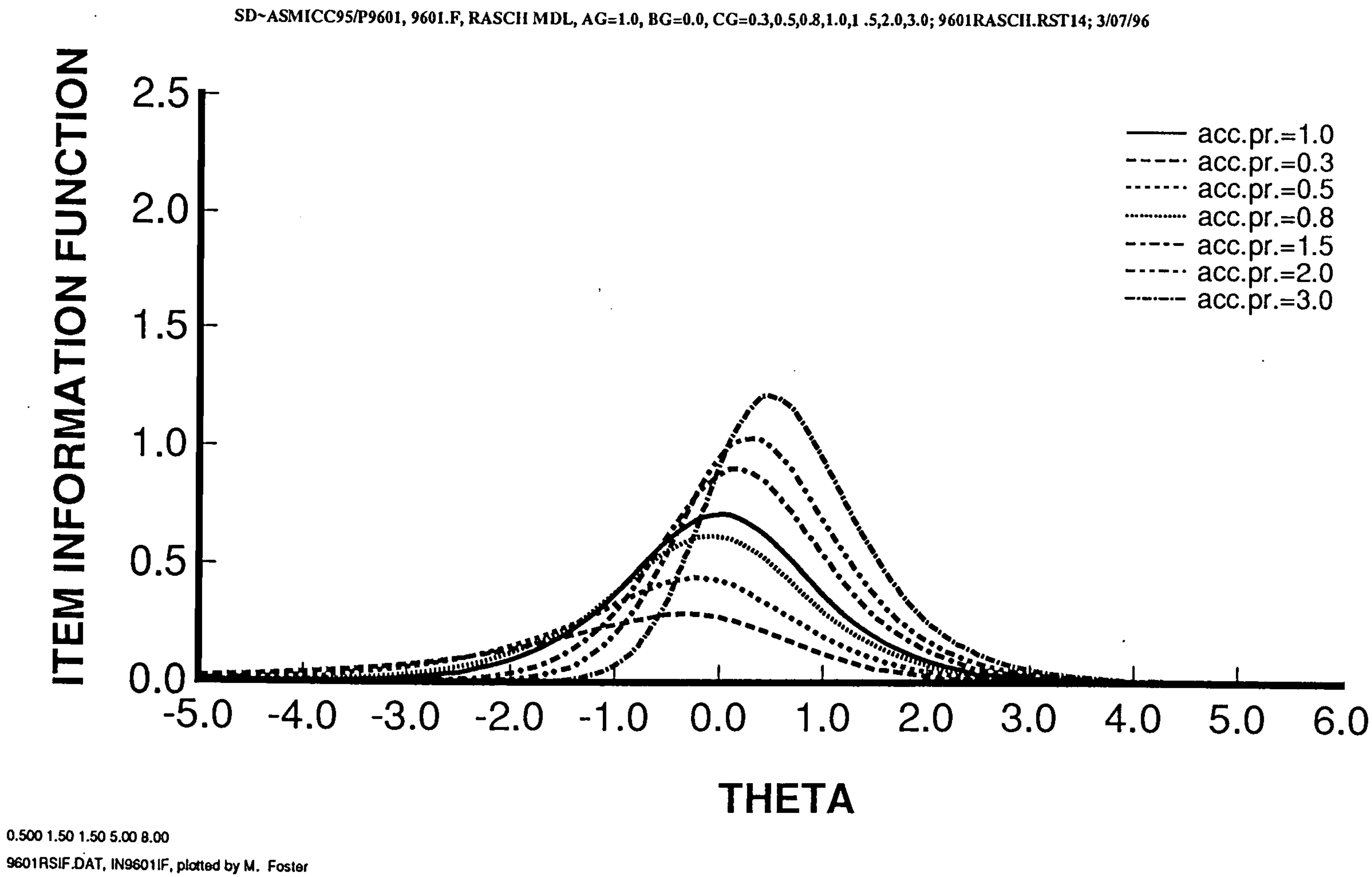
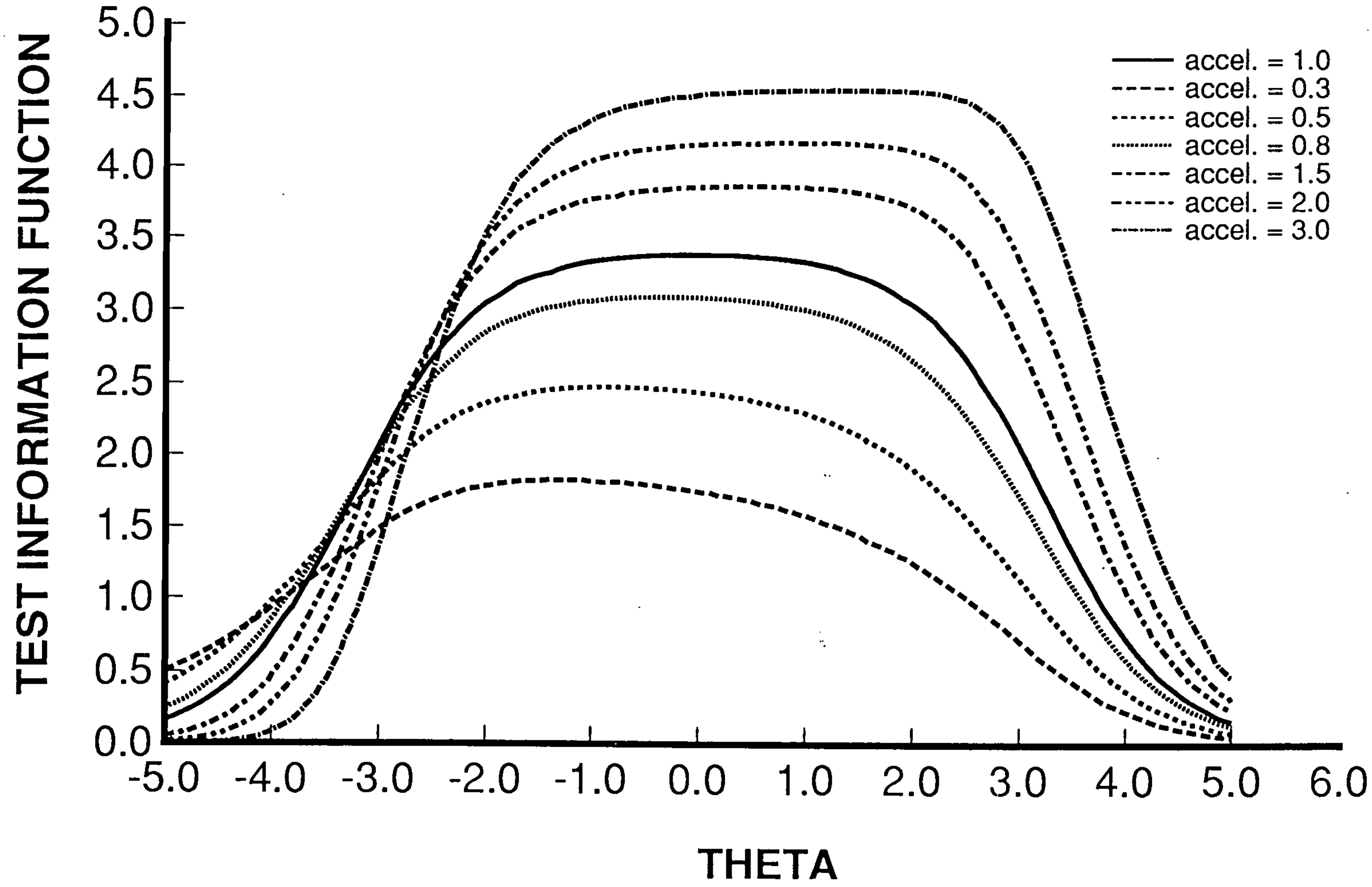


FIGURE 2

Item information functions of the seven items whose item characteristic curves are shown in Figure 1.

SD~AERA/Y1999/P9601, 9601.F, N=13, TEST INF.FUNC.; AG=1; BG=-3,+3: STEP=0.5;CG=0.3,0.5, 0.8,1.0,1.5,2.0,3.0;
9601RASCH*.RST4:*=03 ,05,08,10,15,20,30; 02/11/99

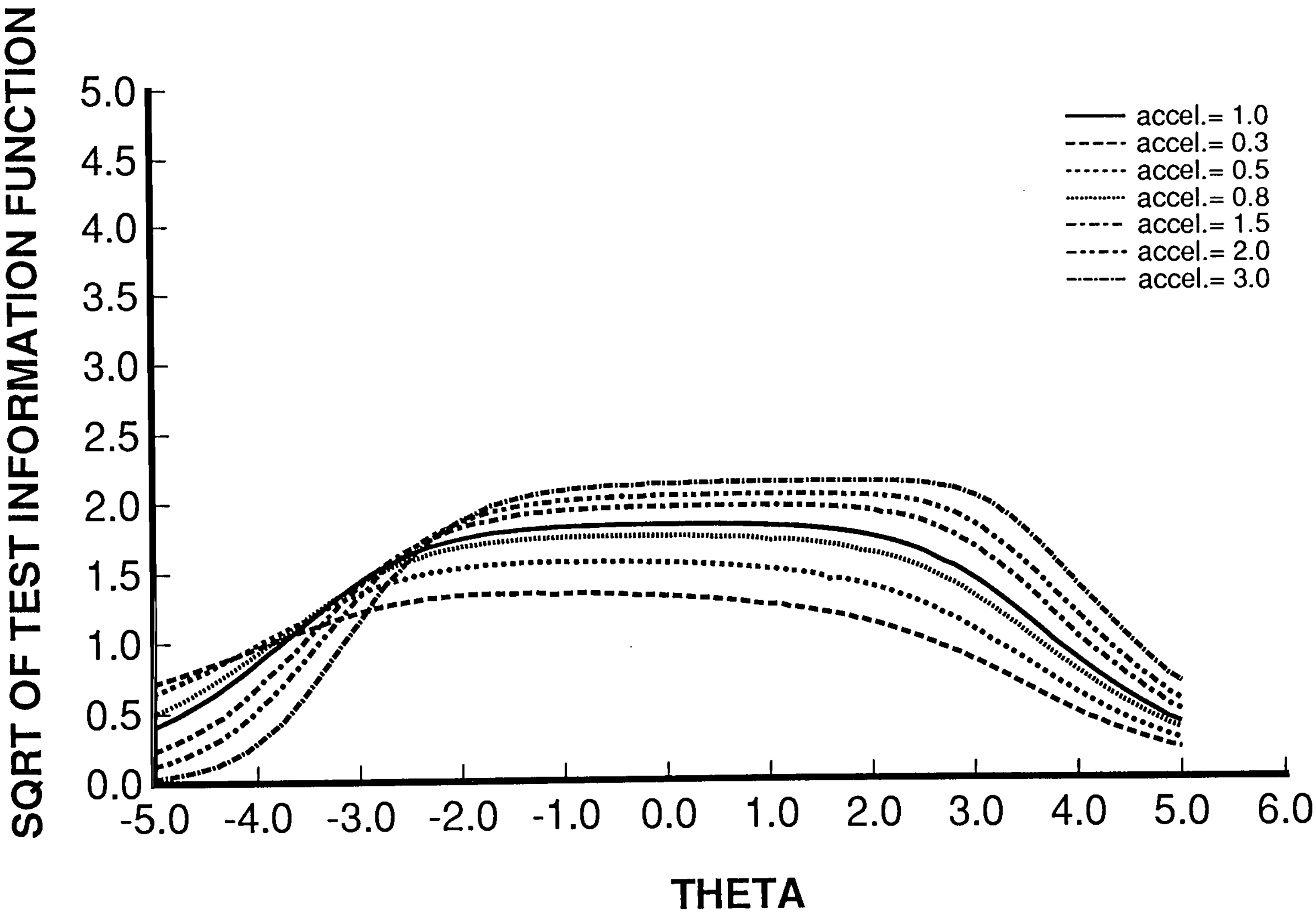


0.600 0.50 1.50 6.50 8.00
9601TI13.DAT, IN9601TI.RP, plotted by R. Pfeil

FIGURE 3

Test information function of seven hypothetical tests of 13 items each, following LPEF models with $\xi_g = 0.3, 0.5, 0.8, 1.0, 1.5, 2.0, 3.0$, respectively, with the common discrimination parameter $a_g = 1$ and the common set of 13 difficulty parameters, $b_g = -3.0, -2.5, -2.0, -1.5, -1.0, -0.5, 0.0, 0.5, 1.0, 1.5, 2.0, 2.5, 3.0$.

SD~AERA/Y1999/P9601, 9601.F, N=13, SQUARE ROOT OF TEST INFORMATION FUNCTION; AG=1,BG=-3,+3:STEP=0 .5;
CG=0.3,0.5,0.8,1.0,1 .5,2.0,3.0; 9601RASCH*.RST4: *=03,05,08,10,15,20, 30; 2/08/99



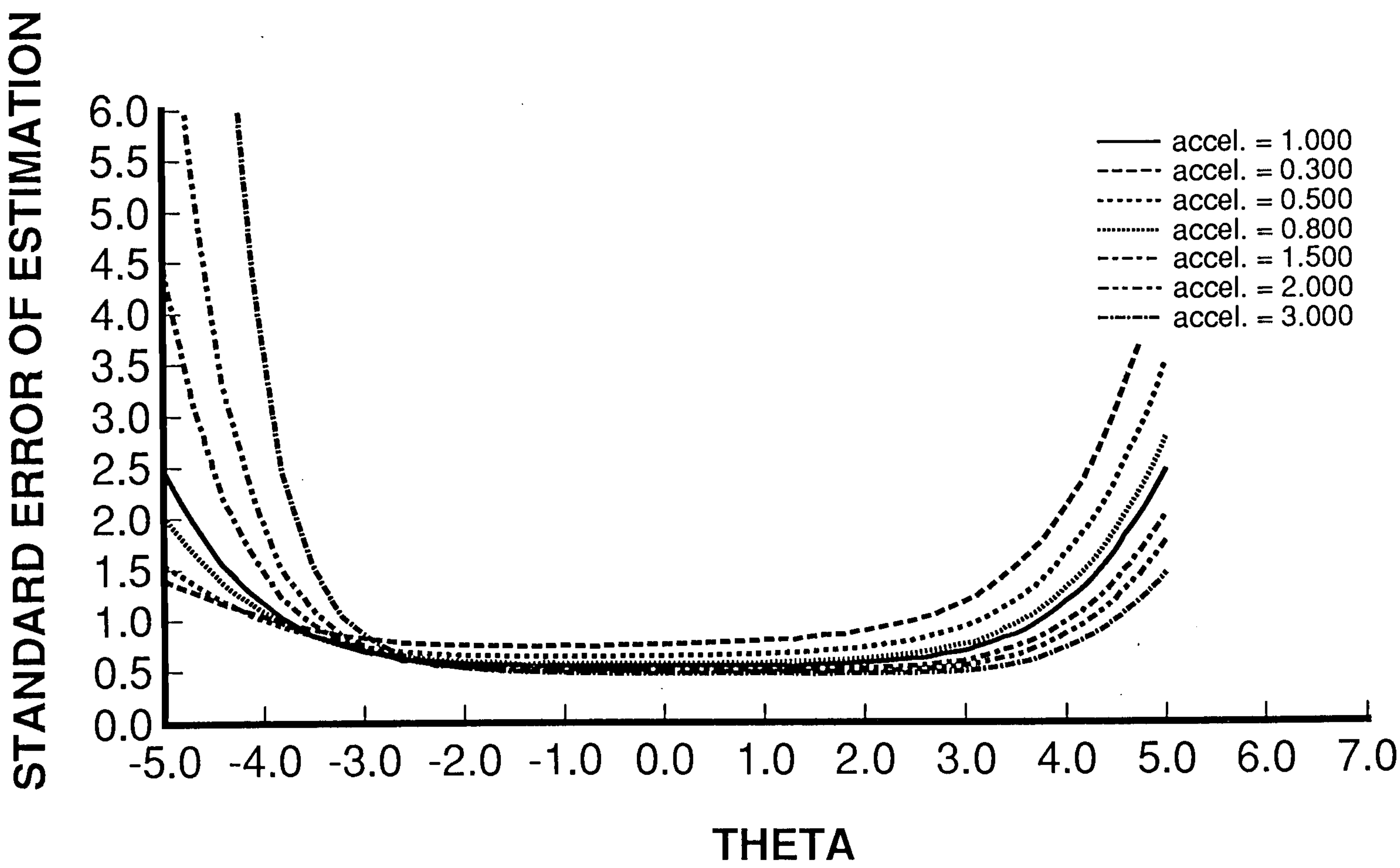
0.600 0.50 1.50 6.50 8.00

9601ST13.DAT, IN9601ST.JC, plotted by J. Chan

FIGURE 4

Square roots of the test information functions of the seven hypothetical tests of 13 items each, following LPEF models whose item parameters are the same as those described in Figure 3.

SD~AERA/Y1999/P9601, 9601.F, N=13, LOCAL STANDARD ERROR OF ESTIMATION; AG=1.0, BG=-3, +3:STEP 0.5, CG=0.3,0.5,0.8,1.0,1 .5,2.0,3.0;
9601RASCH#.RST4: {#=03,05,08,10,15,20 ,30}, 2/11/99



0.500 1.00 1.21 6.00 8.00
9601SE13.DAT, IN9601SE.JC, plotted by J. Connelly

FIGURE 5

Local standard errors of estimation of the seven hypothetical tests of 13 items each, following LPEF models whose item parameters are the same as those described in Figure 3.

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